

Logical Consequence

Introduction

- talking about the second Chapter in Steven Read's book "Thinking about Logic", in this Chapter he investigates the main concept of logic, the Logical Consequence
- as heard before, classical logic is a matter of form: individual arguments are valid only in virtue of instantiating valid logical forms
 - that means however the schematic letters are interpreted, truth is preserved
 - so in general an argument is invalid if there is some substitution for the descriptive terms in it which yields true premisses and a false conclusion; and an argument is valid if there is no such substitution
 - the substitutional criterion over- and undergenerates (that is, in which it counts as valid inferences which are clearly invalid and the other way round)
- the classical account as deriving from the work of tarski has been amended to avoid such overgeneration by adding to the substitutional account a varying domain of interpretation as presented by Mr Wojcicki

Compactness

- next aspect is where classical account undergenerates:
- we say that a consequence relation is compact if any consequence of an infinite set of propositions is a consequence of some finite subset of them
classical logical consequence is compact, this means
an inference is valid if and only if the conclusion follows from a finite set of premisses
- the limitation on expressibility follows from choosing a compact logic (classical logic is compact for historic reasons, as logicians tried to axiomatize mathematics)
tried to find a finite set of axioms from which the whole of mathematics could be defined
- famous example is the ω -rule (ω is the standard model of arithmetic):
some formula A is true for every natural number, $0,1,2 \dots$
so $A(0)$ is true, $A(1)$ is true etc.
it obviously follows that the formula “for every $n \in \mathbb{N}$, $A(n)$ is true
→ logical consequence of $A(0)$, $A(1)$ and so on
but this consequence does not follow from a finite subset of that collection, it would require that a proof be an infinite object
so the validity of the ω -rule is not accepted in orthodox classical consequence (so it undergenerates here)
- Goedel showed up in 1931 that no compact logic can do this. in a non-compact logic there can be a categorical set of formulae for arithmetic, but proof methods (as usually understood) require compactness.
- at this point Read does an excursion about first- and second-order logic. I skip this as it would take too long and would be very detailed, as a summary one could say that second-order logic allows to range quantifiers over and refer to properties.
- back to the ω -rule:
we might propose that the ω -rule as it stands is invalid, it needs an extra premiss: “and these are all the numbers“
this response is inadequate as a defence of classical logic and its compactness, as this extra-clause cannot be expressed in first-order logic (and if we see it that way, we cannot even express the ω rule in first-order terms)
- Read’s conclusion is that:
 - first-order logic/classical logic is complete in the unusual sense that there is a set of inference rules by which a proof can be constructed deriving every first-order consequence from given premisses
 - it is incomplete in that not every intuitively valid consequence of these premisses is indeed a first-order consequence of them (so the classically consequence undergenerates here in the usual meaning)
 - second-order logic is complete in the sense that its consequence relation matches the intuitive one
 - and it is incomplete that there is no set of proof-methods and inference rules adequate to derive all those formulae from the premisses whose consequences they are

Matter and Form

- we have now a clear conception about the account of logical consequence
as said it is a matter of form, an argument is valid if it instantiates a valid form; and a form is valid if there is no permissible interpretation of the descriptive terms under which the premisses are true and the conclusion false.

here he gives an example:

“nothing is both round and square“

it's just not true that it could be false because one can replace “round“ and “square“ by suitable expressions and obtain a proposition that is false

so varying the interpretation cannot show the failure of an entailment

- the interpretational account, as represented by the classical criterion is an attempt to avoid certain modal metaphysics.

(i will not discuss them in detail)

Read shows that this reduction fails, as seen by the round-and-square example. trying to overcome this problem with the “varying domain“ account seemed to be good strategy.

but with making a subst one looked to see if the proposition was actually true or false, and when varying the domain, one has to see if the proposition would be true or false. so modality has returned.

he says that we cannot vary the domain, he states that “logical consequence is really a matter of what would be the case if the premisses were true.“

as he further says, classical logic tries to replace the modal talk about necessity and impossibility by interpretations and subst. but this is inadequate.

by insisting that all logical consequence is a matter of form, it fails to include as valid consequences those inferences whose correctness depends on the connections between non-logical terms.

such connections are valid in a virtue of matter, not of form. these connections are often called analytic connections

to defend the classical logic one might dismiss analytic connections as lacking the firmness of logic, or to dismiss them as not logical truth, but truth about meaning.

another example he gives is “every plane map can be coloured with at most four colours“; through true, and necessarily true, its not a logical truth but a truth of mathematics.

so not all necessary truths are truths of logic.

Reads conclusion is that first-order classical logic is inadequate to describe all valid consequences. whether the theories which complete it should be called logic, or whether they are substantial theories - of mathematics, or of meaning, is arguable.

- in the next paragraph he talks about relevance, instructed to skip it as it criticizes classical logic